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Improved Solution on Students Answer Sheet Assessment Using Fuzzy Rules

Bhavin Kakani[1], Dishank Dalal[2], Abhishek Dabhi[3]
Electronics & Communication Engineering, Institute of Technology, Nirma University
[1] bhavin.kakani@nirmauni.ac.in, [2] 13bec021@nirmauni.ac.in, [3] 13bec020@nirmauni.ac.in

***Abstract*— From a wide variety of applications in the areas of engineering and financial sector, Fuzzy logic techniques has a potential to model academic performance evaluation too. Manual errors in student evaluation can affect their present and future opportunities, so there arises a need to develop a model that can help teachers to fairly evaluate student's performance. This article presents an improved solution to the existing evaluating methods by taking degree of confidence and satisfaction levels of evaluators into consideration using fuzzy logic.**

***Index Terms*— Fuzzy Logic, Degree of Confidence, Degree of Satisfaction, Fuzzification, Defuzzification, Linguistic variable, Optimism Index**

I. INTRODUCTION

The fuzzy set theory was introduced in 1965 by Lotfi A. Zadeh and since then it has been extensively used in resolving problems in numerous fields and education evaluation is one of them. Bai and Chen in [1] offered a method for automatically constructing grade membership functions of lenient type grades, strict type grades and normal type grades assigned by teachers. Centered on this constructed grade membership function, system uses fuzzy logic to understand the marks of students. It provides a valuable approach to evaluate student's answer sheets in a smoother and unbiased manner. Then Biswas in [2] presented a method for fuzzy evaluation by applying fuzzy sets in student's answer sheets. The limitations of this method were that it was a time consuming method and same grade can be assigned to two different fuzzy marks which was based for students' evaluation.

Then Chen and Lee in [3] offered methods to apply fuzzy sets in students' answer scripts which were used to overcome these limitations. This method was faster and accurate but it too had a drawback. It was unable to handle the condition where values were represented by fuzzy numbers associated with degree of confidence without considering optimism index of the evaluator while evaluating. Then Chen and Wang in [4] gave a new method for assessing students answer scripts centered on interval valued fuzzy mark sheets. The optimism index of evaluator was considered. Further, Hui-Yu Wang and Shyi-Ming Chen in [5] proposed a new method for evaluation using fuzzy numbers associated with degree of confidence of the evaluator. The satisfaction levels were represented by fuzzy numbers associated with degree of confidence lying between 0 and 1. Then Saleh and Kin in [6] presented a method using fuzzy logic for evaluation of students' answer sheets which considers the difficulty, importance and complexity of questions and then fuzzify and defuzzify it. This presented a transparent and easy method of evaluation. It convinces students who are not contented with their assessment. The limitation of this system is in deciding the values of complexity and importance. Then in [7] Li and Chen gave the method for evaluation where the accuracy rate, time rate, difficulty and importance are produced automatically using fuzzy logic.

Nolan also presented in [8] the strategy and development of a scoring system. The main function of this system is to help examiners evaluating in the less time and accurately. Then Wang and Chen then in [9] presented a novel method for evaluation using vague values. Each question was given a vague mark which was a vague set representing vague value. Then in [10] Bardul presented a method to evaluate the performance of the students as single and as a group. The main idea of this analysis was to improve the surviving fuzzy attitude used in evaluation. Thus the main attention of this theory was student's answer sheet assessment and also student's group assessment.

After that Wang and Chen presented new approach in [11] for evaluating the answer sheets of students in which values are characterized by fuzzy numbers and also optimism index of the evaluator is used to denote the type of the evaluator. This method covered many limitations of previous methods. This method can assess the answer sheets more fairly and clearly.

The study of student's evaluation done by Chih-Hsun Hsieh in [12] converts linguistic variables into fuzzy triangular numbers and uses the function principle in place of the extension principle to find the student's marks. This reduces the problems faced while undergoing operations. Then Bardul and Mahamad in [10] gave the method that represents some satisfaction levels by normalized values. They use fuzzy numbers to obtain consistent fuzzy scores which in turn calculate the total marks. This method can provide more efficient and better information showing student's result.

II. NEED OF FUZZINESS IN EVALUATION

Student's evaluation is also affected by the number of evaluators/examiners. As the number of examiners increases it becomes hard to apply the same checking process for all the students. Few observed barriers in this regards are: (1) evaluator's level of satisfaction and degree of confidence. (2) Evaluation rubrics are different for evaluators (3) personal factors like fatigue, stress etc. So, Fuzziness and vagueness are the features that generates problem in making decisions. Subjective evaluation gives place to difference in opinion resulting in Fuzziness. Fuzzy logic thus helps to sum up the data and focus on decision related information. The two major concepts of fuzzy logic which has critical role in its applications are of a linguistic variable and of fuzzy if-then rule.

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III. FUZZY SYSTEM

Fuzzy logic is used to handle the reasoning that is fuzzy rather than exact. It can deal with the concept of partial truth where the values may lie between fully true and fully false. Fuzzy system has four major parts. They are fuzzification, knowledge base, decision making logic and defuzzification. The transformation of input crisp values into fuzzy values i.e. membership grades for linguistic variables of fuzzy sets is known as fuzzification. This is done by a membership function which assigns a grade to each linguistic word [1]. Knowledge base contains important definitions which are used in control rules and manipulating data. It also defines the control scheme and objectives by means of linguistic control rules. Decision making logic directs the human decision using fuzzy concepts and fuzzy rules. Thus fuzzy set obtained after composing the rules is converted into a crisp value. This is called defuzzification.

IV. PROPOSED METHODOLOGY

A. Calculations of evaluators' optimism index

Different evaluators possess different characteristics during evaluation. One may be strict then other may be lenient. To formulate this an index of optimism (λ) where $\lambda \in (0, 1)$ denotes the degree of optimism of the evaluator is proposed. It shows the nature of the evaluator. If $0 \leq \lambda \leq 0.5$ then the evaluator is considered strict, if $\lambda = 0.5$ evaluator is normal and if $0.5 < \lambda \leq 1$ then the evaluator is lenient.

Let us suppose a case where 8 numbers of evaluators who are given 6 answer sheets to evaluate according to them. Consider that they assign the following marks as shown in table 1 shown below.

TABLE 1
MARKS OF STUDENTS EVALUATED BY DIFFERENT TEACHERS

	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8
M_1	42	24	27	31	34	39	43	44
M_2	32	35	36	40	43	43	52	54
M_3	47	54	66	70	72	72	72	73
M_4	69	70	75	76	81	83	85	89
M_5	81	85	85	92	92	94	96	56
M_6	55	61	75	96	64	95	74	42

STEP 1) Firstly the mean of marks is obtained of each 6 students answer scripts using the general formula

$$\text{Mean } M_i = \frac{\sum_{j=1}^{x} M_{ij}}{x}$$

Where 'x' is the number of evaluators and 'y' is the number of students answer scripts and i = 1, 2… y and j =1, 2… x.
Thus we get the following results listed in table 2:

TABLE 2
MEAN OF MARKS OF EACH STUDENT

	M_1	M_2	M_3	M_4	M_5	M_6
Mean M	32.6	41.8	65.7	78.5	90.6	68.8

STEP 2) Then the mean of marks assigned by every 8 evaluators is calculated. This will give every single evaluator's mean. We use the following formula.

$$\text{Mean } N_j = \frac{\sum_{i=1}^{y} M_{ij}}{y}$$

Where i = 1, 2… y and j =1, 2… x.
Thus we get the following results stated in table -3:

TABLE 3
MEAN MARKS ASSIGNED BY EACH EVALUATOR

	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8
Mean N	49.3	54.8	60.6	67.5	64.3	71	70.3	59.6

STEP 3) Now, The mean of mean marks of students is obtained by dividing the sum of mean of marks of 6 students by total number of students answer scripts.
Thus we get using formula

$$M' = \frac{\sum_{i=1}^{y} \text{Mean } M_i}{y}$$

Thus here we get $M' = 378/6 = 63$.

STEP 4) Now we find the difference between Mean N_j and M'. We use the formula

$$d_i = \text{Mean } N_j - M'$$

Where j =1, 2… x.
Here we get the following table 4:

TABLE 4
DIFFERENCE CALCULATION

	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8
d_j	-12.5	-8.2	-2.4	4.5	1.3	8	7.3	-3.4

Now to calculate the optimism index we use Mamdani's fuzzy interference method [13][14]. We do fuzzification of Input by using triangular membership function and defuzzification by using Center of gravity method (COG). Here we use difference (d_j) of every evaluator as input variable 'differ'. We use the following triangular fuzzy number representation of fuzzy sets to fuzzify the input crisp value.

1) Very Large Negative Value (LVN) =(-17 -15 -13)
2) Large Negative (LN) = (-14 -12 -10)
3) Medium Negative (MN) = (-11 -9 -7)
4) Small Negative (SN) = (-8 -6 -7)
5) Very Small Negative (VSN) = (-5 -3 -1)
6) Neutral (N) = (-2 0 2)
7) Very Small Positive (VSP) = (1 3 5)
8) Small Positive (SP) = (4 6 8)
9) Medium positive(MP) = (7 9 11)
10) Large Positive(LP) = (10 12 14)
11) Very Large Positive(VLP) = (13 15 17)

Now, we find the type of the evaluator using this difference. Thus we will get the value of optimism index as the output.

We form following triangular functions representing fuzzy sets.

1) Very Very Strict (VVS) = (0 0 0.1)
2) Very strict (VS)= (0 0.1 0.2)
3) Strict (S) =(0.1 0.2 0.3)
4) Not much strict (NMS) = (0.2 0.3 0.4)
5) Less strict (LS) = (0.3 0.4 0.5)
6) Normal (NM) =(0.4 0.5 0.6)
7) Less strict(LS) = (0.5 0.6 0.7)
8) Not much strict(NMS) =(0.6 0.7 0.8)

9) Lenient (L) = (0.7 0.8 0.9)
10) Very Lenient (VL) = (0.8 0.9 1)
11) Very Very Lenient (VVL) = (0.9 1 1)

In the next step we form the fuzzy rules to associate the input to the output. Following are the 11 fuzzy rules formed:

1) If differ is N then optimism index is NM.
2) If differ is VSN then optimism index is LS.
3) If differ is VSP then optimism index is LL.
4) If differ is SN then optimism index is NMS.
5) If differ is SP then optimism index is NML.
6) If differ is MN then optimism index is S.
7) If differ is MP then optimism index if L.
8) If differ is LN then optimism index is VS.
9) If differ is LP then optimism index is VL.
10) If differ is VLP then optimism index is VVL.
11) If differ is VLN then optimism index is VVS.

So, after fuzzification, using rules and defuzzification process using MATLAB we get the values of optimism index (λ) as shown below in table 5.

TABLE 5
OPTIMISM INDEX OF EACH EVALUATOR

	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8
λ	0.1	0.2	0.4	0.65	0.529	0.8	0.73	0.4

B. *Evaluation considering degree of confidence*

We proposed a method which evaluates students' answer sheets considering degree of confidence and satisfaction level of the evaluator. Degree of confidence shows how confident the examiner is while assigning the marks. Satisfaction level shows how much satisfied the examiner is with the answer given by the student.

We obtain the satisfaction levels awarded to each questions by the expected truth values of each unclear satisfaction values. He assigns satisfaction level of answer to every question instead of marks using given vague grade sheet. Degree of confidence connected with satisfaction level is used to estimate α cut which consecutively is used to find the total marks of every student. Degree of confidence of examiner is used to form the fuzzy interval. Then the system transforms vague marks into fuzzy marks. Then using this fuzzy marks we calculate the total marks of every student.

The inputs for this system is the answer sheet of a student, a vector representing the maximum marks of each question, satisfaction level given in table (1) and the optimism index (λ) of the evaluator. Then after following the given procedure we get the output as the total marks obtained by the student considering degree of confidence and satisfaction level of the examiner.

STEP 1) Let us consider that maximum marks in an examination are 100 and total number of question to be answered is 4 and each question carries 25 marks. Let us assume that the optimism index of the evaluator who is evaluating all the answer sheets is 0.7. This is a case of single evaluator who assigned following marks to the student as shown in table 6.

TABLE 6
MARKS GIVEN BY EVALUATOR

Question No.	Marks assigned
1	20
2	18
3	21
4	15

TABLE 7
SATISFACTION LEVEL AND CORRESPONDING VAGUE VALUE

Satisfaction levels	Vague satisfaction values
Extremely Good	[1 ,1]
Very Very Good	[0.90 ,0.99]
Very Good	[0.80 ,0.89]
Good	[0.70 ,0.79]
Less Good	[0.60 ,0.69]
Fair	[0.50 ,0.59]
Less Bad	[0.40 ,0.49]
Bad	[0.25 ,0.39]
Very bad	[0.10 ,0.24]
Very Very bad	[0.01 ,0.09]
Extremely bad	[0 ,0]

The equivalent vague satisfaction values for different satisfaction level is given below in table 7. And the vague mark represented by vague values of the questions in a vague grade sheet is shown in table 8.

TABLE 8
VAGUE GRADE SHEET

Question	1	2	3	4
DOC	0.75	1	0.75	0.95
EG	[0.8,0.9]	[0,0]	[0,0]	[0,0]
VVG	[0.9,0.95]	[0,0]	[0,0]	[0,0]
VG	[0,0]	[0,0]	[0.85,0.9]	[0,0]
G	[0,0]	[0.6,07]	[0.75,0.8]	[0,0]
MG	[0,0]	[0.9,0.95]	[0.5,0.6]	[0,0]
F	[0,0]	[0.55,0.6]	[0,0]	[0,0]
MB	[0,0]	[0,0]	[0,0]	[0,0]
B	[0,0]	[0,0]	[0,0]	[0.5,0.6]
VB	[0,0]	[0,0]	[0,0]	[0.9,0.95]
VVB	[0,0]	[0,0]	[0,0]	[0.2,0.4]
EB	[0,0]	[0,0]	[0,0]	[0,0]

STEP 2) Calculate the corresponding expected truth value E(X) of every satisfaction level X in the vague answer sheet shown in table 8 using the formula

$$E(X) = (1-\lambda)*a_x + \lambda(1-b_x) \quad \text{(i)}$$

Where λ is the optimism index of the evaluator and $[a_x \ b_x]$ is the vague satisfaction value. The value of optimism index of the evaluator is given to be 0.7.

We get the following table 9 using formula (i)

TABLE 9
EXPECTED TRUTH VALUE

Satisfaction levels	Vague satisfaction level	E(X)
Extremely Good (EG)	[1 ,1]	1.0
Very Very Good(VVG)	[0.90 ,0.99]	0.963
Very Good(VG)	[0.80 ,0.89]	0.863
Good (G)	[0.70 ,0.79]	0.763
Less Good(LG)	[0.60 ,0.69]	0.663
Fair (F)	[0.50 ,0.59]	0.563
Less Bad(LB)	[0.40 ,0.49]	0.463
Bad (B)	[0.25 ,0.39]	0.348
Very Bad(VB)	[0.10 ,0.24]	0.198
Very Very Bad(VVB)	[0.01 ,0.09]	0.066
Extremely Bad (EB)	[0 ,0]	0

STEP 3) Now we calculate the expected truth value E (X_k) of every vague truth value X_k in the vague sheet shown in table (2) using the same formula (i), where E (X_k) ϵ [0, 1] and 1 ≤ k ≤ 1.
We get the following table 10.

TABLE 10
CALCULATION OF EXPECTED TRUTH VALUE

Question	1	2	3	4
DOC	0.75	1	0.75	0.95
EG (E(X_1))	0.87	0	0	0
VVG (E(X_2))	0.935	0	0	0
VG (E(X_3))	0	0	0.885	0
G (E(X_4))	0	0.67	0.785	0
MG (E(X_5))	0	0.937	0.57	0
F (E(X_6))	0	0.585	0	0
MB (E(X_7))	0	0	0	0
B (E(X_8))	0	0	0	0.57
VB (E(X_9))	0	0	0	0.935
VVB (E(X_{10}))	0	0	0	0.34
EB (E(X_{11}))	0	0	0	0

STEP 4) Now we find the degree of satisfaction D (qi) of each question qi of the students' answer sheet using the formula

$$D(qi) = \frac{E(X_1)*E(EG)+E(X_2)*E(VVG)+\cdots\ldots\ldots E(X_{11})*E(EB)}{E(X_1)+E(X_2)+\cdots\ldots+E(X_{11})} \quad \text{(ii)}$$

Where $E(X_i)$ is the estimated satisfaction value of vague satisfaction value X_i and i ∈ [1, 11] and 0 ≤ D(qi) ≤1. Large value of D(qi) indicates high degree of satisfaction showing that the evaluator is contented with the answer of question qi.
The degree of satisfaction calculated by formula (ii) is shown in the table 11 given below:

TABLE 11
CALCULATION OF DEGREE OF SATISFACTION

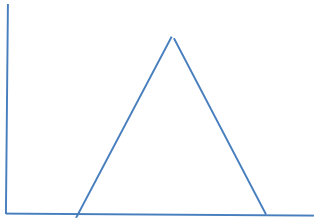
Question	1	2	3	4
DOC	0.75	1	0.75	0.95
EG (E(X_1))	0.87	0	0	0
VVG (E(X_2))	0.935	0	0	0
VG (E(X_3))	0	0	0.885	0
G (E(X_4))	0	0.67	0.785	0
MG (E(X_5))	0	0.937	0.57	0
F (E(X_6))	0	0.585	0	0
MB (E(X_7))	0	0	0	0
B (E(X_8))	0	0	0	0.57
VB (E(X_9))	0	0	0	0.935
VVB (E(X_{10}))	0	0	0	0.34
EB (E(X_{11}))	0	0	0	0
Degree Of Satisfaction	0.9808	0.6668	0.7777	0.22

Now, we match the satisfaction level E(X) from table (3) corresponding to the degree of satisfaction. We get VVG corresponding to 0.9808, LG corresponding to 0.6668, G corresponding to 0.777 and VB corresponding to 0.2200.

STEP 5) Next we find α-cut of each satisfaction level based on degree of confidence of the examiner. We use the formula for α-cut given as

$$\gamma_{A=}\ [a-(1-\gamma)\alpha, a+(1-\gamma)\beta] \quad \text{(iii)}$$

Where 'a' is the maximum value, α and β are as shown in figure and γ is the degree of confidence.



Thus for VVG and 0.75 degree of confidence, we have a = 0.963. Using formula (iii) and values of α, β from table (3) we get α-cut as [0.94725, 0.96975]

For LG and degree of confidence 1.0, we have a = 0.663 and we get α-cut = [0.663, 0.663]

For G and degree of confidence 0.75, we have a=0.763 and we get α-cut = [0.747225, 0.81025]

For MG and degree of confidence 0.95, we have a= 0.198 and we get α-cut = [0.1931, 0.2001]

Thus for obtaining fuzzy marks, we multiple by 100 and we get the following values of α-cut of each fuzzy marks based on degree of confidence.

(VVG) 0.75 = [94.7 96.9]
(LG) 1.0 = [66.3 66.3]
(G) 0.75 = [74.7 81.0]
(VB) 0.95 = [19.3 20.0]

STEP 6) Now we find the interval valued mark $[x_{i1}, x_{i2}]$ of every question (qi) using the formula given as

$$[x_{i1}, x_{i2}] = \frac{s_i}{s_1+s_2+\cdots\ldots s_n} * [b_{i1}, b_{i2}] \quad \text{(iv)}$$

Where $[b_{i1}, b_{i2}]$ is the value of α-cut found in the previous step, s_i is the marks the student got in the ith question and $s_1, s_2, s_3, , , , s_n$ are the maximum marks of n questions.
We know n=4, maximum marks of each question is 25 and s_1=20, s_2=18, s_3=21 and s_4=15 by table(A). Thus using (iv) we get, $[x_{11}x_{12}]$ = [18.94, 19.38]
$[x_{21}x_{22}]$ = [11.934, 11.934]
$[x_{31}\ x_{32}]$ = [15.687, 17.01]
$[x_{41}x_{41}]$ = [2.895, 3]

STEP 7) Now we defuzzify the marks of each question qi using optimism index by using the formula.

$$q_i = (1-\lambda) * x_{11} + \lambda x_{12} \quad \text{(v)}$$

Then total marks (T) of every student is calculated using the formula

$$T = \sum_{i=1}^{4} q_i \quad \text{(vi)}$$

Thus we get
T = (0.3)(18.94)+0.7(19.38)+(0.3)(11.934)+0.7(11.934)
+ (0.3)(15.687)+0.7(17.01)+(0.3)(2.895)+0.7(3)
= 50.76.

Thus we form the fuzzy mark sheet showing satisfaction levels considering degree of confidence.

TABLE 12
FUZZY MARK SHEET

Question No	Satisfaction levels	DOC
1	VVG	0.75
2	EG	1.0
3	G	0.75
4	VB	0.95
-	**Total Marks**	50.76

V. CONCLUSION

We conclude that the system introduced in the first section is very helpful in studying student's results. As students are futures of our country. Thus we should keep a close watch on their performance. Their bright future guarantees the brilliant future of their country. Thus with the help of the above defined method we can know about the students who performed poor in the examination. So, we can apply improvement methods on them. The Institution or the University itself decides that what measures they should take in order to uplift the result of their students. Moreover, this method is easy and can be understood by all. The result shown as an output is exact and impartial. This is time saving and depicts the result of the student without any error.

In case of evaluation by many evaluators, there is a chance that some of the examiners are strict, lenient or normal type. Thus in the second section, we get a method to find out the type of evaluator using MATLAB. Many areas of evaluation use the value of this optimism index for further analysis. Hence by the above mentioned method we can evaluate the more exact value so that student's answer sheet can be corrected fairly and transparently. And in many researches it is shown that keeping degree of confidence and associated satisfaction level same but different optimism index have a significant effect in the result. Thus optimism index (λ) of the examiner cannot be ignored. If the value of λ lies between 0 and 0.5 then the examiner is strict, if λ is equal to 0.5 then the examiner is normal type and if the value of λ lies between 0.5 and1 the examiner falls in lenient category.

In the last section student's marks are evaluated considering degree of satisfaction associated with the satisfaction. In this way a new fuzzy mark sheet of every student will generate. It will depict extra information of students' performance in giving answers of every examination question. Rather this technique will also be useful to compare results of those students who have same linguistic terms. Moreover, the information can help student, teachers and also other authorities to know about the all over view of students' performance. It is also not time consuming as evaluating students' answer sheets is very tedious and monotonous. But the method discussed above is time consuming and evaluate in an intelligent and fair manner

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